Gas Turbine Cycle

Lecture 2
Simple gas cycle

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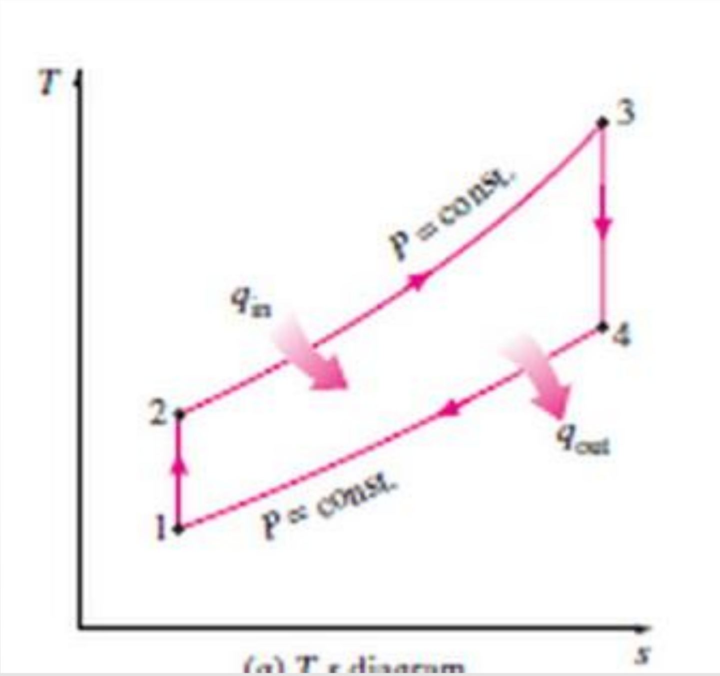
Air standard analysis

Using the perfect gas relations, we can write equations (1 & 3) in team of pressure ratio (r_p) across the turbine:



- pressure ratio (r_p) = $\frac{p_3}{p_4} = \frac{p_2}{p_1}$

Using the isentropic relations :-

The absolute temp ratio across the compressor:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = (r_p)^{\frac{k-1}{k}} \quad \text{......................(4.a)}$$

Where: k = $\frac{c_p}{c_v}$ = 1.4 (for cold air)

And across the turbine is give as :-

$$\frac{T_3}{T_4} = \frac{P_3}{P_4}^{\frac{k-1}{k}} = (r_p)^{\frac{k-1}{k}} \quad \text{.......................(4.b)}$$

k = 1.33 (for hot gas)

So that the specific work of the turbine from eq.3 is given as :-

$$W_T = C_p T_3 \left(1 - \frac{T_4}{T_3}\right) = C_p T_3 \left(1 - \frac{1}{(r_p)^{\frac{k-1}{k}}}\right) \quad \text{.....................................(5)}$$

Where $T_3 \neq T_2$

And the specific work of compressor from eq.1 is given as :-

$$W_c = C_p T_2 \left(1 - \frac{T_1}{T_2}\right) = C_p T_2 \left(1 - \frac{1}{(r_p)^{\frac{k-1}{k}}}\right) \quad \text{.................................(6)}$$

where $(r_p)_c = (r_p)_T = \frac{p_2}{p_1} = \frac{p_3}{p_4} = constant$.

The net specific work of the gas turbine cycle :-

$W_{net} = W_T - W_C$ ……………………………………………………(7)

And the output power, $P = \dot{m} * W_{net}$ ……………………………………(8)

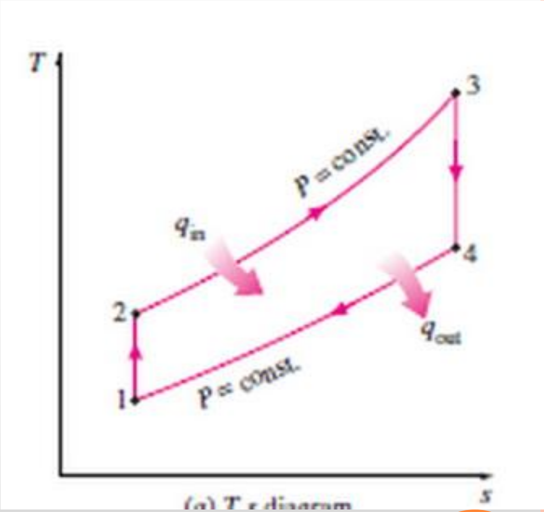
The thermal efficiency of the cycle is:

$$\eta_{th} = \frac{W_{net}}{Q_{add}} = \frac{W_T - W_C}{Q_{add}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

$$= \frac{C_p(T_3 - T_4) - C_p(T_2 - T_1)}{C_p(T_3 - T_2)} = \frac{C_p(T_3 - T_2) - C_p(T_4 - T_1)}{C_p(T_3 - T_2)}$$

$$= 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{T_1}{T_2}\left[\frac{\left(\left(\frac{T_4}{T_1}\right) - 1\right)}{\left(\left(\frac{T_3}{T_2}\right) - 1\right)}\right], \text{ where } \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

$$= 1 - \frac{T_1}{T_2} = 1 - \frac{1}{\frac{T_2}{T_1}} = 1 - \frac{1}{(r_p)^{\frac{k-1}{k}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$



Cold air standard (constant specific heat) .

$W_c = C_p\,(T_2 - T_1)$

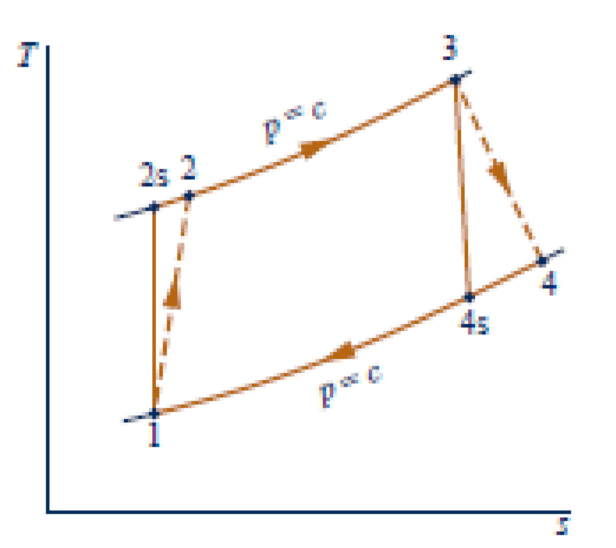
Air – standard (variable specific heat) .

$W_c = h_2 - h_1$

:Example : Gas turbine unit has pressure ratio ($r_p = \frac{6}{1}$) & max temp is 600°C. The isentropic efficiency (η_c = 0.82 , η_T = 0.85). Calculate the power output in (kW), the flow rate of the hot gasses entering the turbine at rate of 15 $\frac{kg}{s}$. The air entrance temp to the compressor is 15 °C, and find BWR.

for air : $C_p = 1.005 \frac{kJ}{kg.K}$, k = 1.4 .

For gas : $C_p = 1.13 \frac{kJ}{kg.K}$, k = 1.33 .



Solution:

For isentropic compression process (compressor):

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

T_1 = 15+273 = 288 K

$$\therefore T_{2s} = 288 * (6)^{\frac{1.4-1}{1.4}} = 481\ K$$

$$\eta_C = \frac{ideal\ work}{actual\ work} = \frac{T_{2s}-T_1}{T_2-T_1} = 0.82$$

$T_2 = 521$ K

- for isentropic expansion process (turbine):

$$\frac{T_3}{T_{4s}} = \left(\frac{P_3}{P_4}\right)^{\frac{k-1}{k}} = (6)^{\frac{1.33-1}{1.33}}$$, where $T_3 = 600+273 = 873$ K

$T_{4s} = 558$ K

$$\eta_T = \frac{actual\ work}{ideal\ work} = \frac{T_3 - T_4}{T_3 - T_{4s}} = 0.85$$

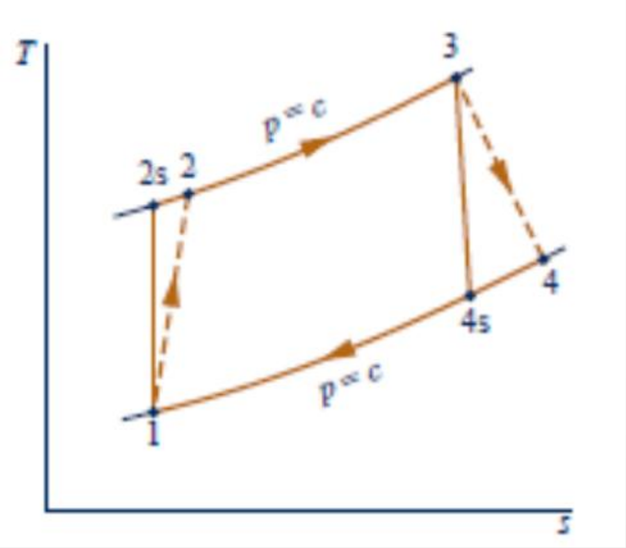
$T_4 = 605$ K

$w_C = C_P\,(T_2 - T_1) = 1.005(521-288) = 234.16$ kJ/kg

$w_T = C_P\,(T_3 - T_4) = 1.11(873- 605) = 297.48$ kJ/kg

$W_{net} = w_T - w_C = 63.31$ kJ/kg

Power output = $\dot{m} * w_{net} = 15*63.31 = 949.72$ kW



$$\eta_{th} = \frac{w_{net}}{q_{add}} = \frac{63.31}{C_p(T_3 - T_2)} = \frac{63.31}{1.11(873-521)} = 15.8\ \%$$

$$BWR = \frac{W_C}{W_T} = \frac{C_p(T_2 - T_1)}{C_p\,(T_3 - T_4)} = 0.787$$